Running Away from the Poor: *Bolsa Familia* and Entry in School Markets

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Abstract

This paper shows that policies designed to reduce inequality may fail because socioeconomic stratification can reappear through the private school system. We develop a theoretical model linking the skill distribution of the student population in the public system to the profitability of private schools. We then use panel data of Brazilian municipalities to examine the relationship between test scores dispersion in the public school system and private school entry. The expansion of the Bolsa-Familia program, an important conditional cash transfer program implemented in Brazil after democratization, is used as a source of exogenous variation for the students' test scores distribution. Our results show that towns where the students' skill distribution widened because of the Bolsa-Familia program were more likely to attract new private schools, which is consistent with the view of endogenous persistence of inequality in the face of a change in political institutions.

Key words: inequality, school market, education, social interactions.
JEL Classification: I21

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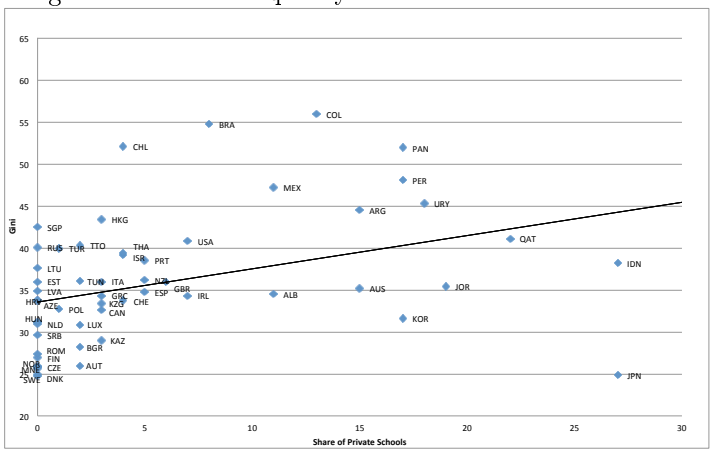
1 Introduction

Economists have always been worried about improving the prospects of the poor and reducing income inequality. One way of doing this is to equalize opportunities, especially through improving access to good schools among the children raised in poor families. Since education is a strong predictor of future earnings, equalizing access to better quality education is likely to increase intergenerational mobility and reduce future inequality. The difficulty with doing this is that individuals and families sort in a number of ways, such as schools and neighborhoods. Poor families tend to live in the same neighborhoods and go to the same schools, generally of lower quality, which reinforces inequality (see Fernandez [16] and Fryer and Katz [21]).

Several policies have been devised and implemented to try to break this cycle, such as the Moving to Opportunity (MTO) experiment (Katz et al [27]), changes in the school finance (Hoxby [26]) and conditional cash transfers programs (CCTs) in developing countries (Saavedra and Garcia [32]). The existing evidence on the results of most of these policies is rather disappointing, however, as improving neighborhoods, sending poor families to richer neighborhoods, or reforming the school finance do not seem to improve education outcomes of poor families per se (see Fryer and Katz [21]). The exception seems to be the CCTs, which have succeed in terms of increasing access of poor children to schools. The case of Bolsa Familia, the CCT program implemented in Brazil after democratization currently reaching 44 million poor people, is seen worldwide as a success case, a classic example of a change in the political system that, by extending the voting franchise, has led to lower poverty and inequality (Meltzer and Richards [30], Acemoglu et al [2]). This has lead to the introduction of conditional cash transfers programs in developed countries as well, as in New York city (Riccio et al [33]). Are conditional cash transfers the way forward to reduce income inequality and increase intergenerational mobility?

This paper shows that the forces at work leading to stratification are harder to overcome than what one might think at first. Policies aimed at equalizing access and quality of the public schools in order to reduce future inequality may fail because stratification can reappear through the private school system. Without the possibility of enrolling their children in a better than average public school, richer families may move their children to private schools. More specifically, we show both theoretically and empirically that stratification through private schools is a function of inequality in the local public education system. Our model maps the distribution of socio-economic characteristics of the student population into the schools' industry organization. Communities that become more unequal (perhaps because of successful inclusion policies) will end up with more stratification across public and private schools in equilibrium, which helps to perpetuate inequality over time. In the empirical part of the paper, we examine what has happened in the Brazilian municipalities after the inclusion of the poor children in schools, brought about by the Bolsa Familia program. We use panel data to estimate the impact of skill dispersion on private school entry, using the change in the share of Bolsa Familia recepients in each municipality

Figure 1: Income Inequality and Share of Private Schools



as an instrumental variable for the grade dispersion in the public school system. Our aim is to understand whether the process of social inclusion brought about the CCTs has increased the probability of private school entry, thereby increasing inequality and stratification within the education system.

Although reliable empirical evidence on the relationship between private schools and inequality is scant, there is a clear correlation between inequality and the share of students in private schools across countries. Figure 1 depicts such a relationship for the countries that participate in the PISA exam.[1] The figure shows that the more unequal countries (mostly the Latin Americans) have private schools. There are countries, however, where a substantial share of the students are in private schools and are not very unequal, like Japan and Korea. Why would this be the case?

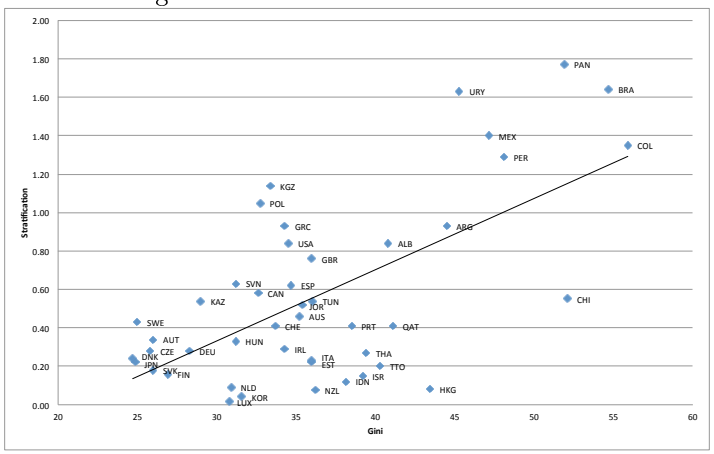
In terms of inequality, more important than the share of students in private schools is how the students are stratified among the public and private schools as a function of their parents economic status. Figure 2 presents the relationship between income inequality and socioeconomic stratification between publicly and privately managed schools, among the countries that do have private schools.[2] It is clear that more unequal countries also tend to have more stratification between students in the public and private schools. Latin American countries, such as Brazil and Colombia, are notable examples of highly unequal countries where the rich attend private schools while the poor have to resort to the lower quality public system.

Our theoretical model consists of a two-stage game where a private school first decides whether to enter the market and then sets its price on the second stage, conditional on entry. Our model builds on Epple and Romano [15], espe-

[1] OECD [31].

[2] Stratification is measured as the difference in the average socioeconomic background of students who attend privately managed schools and those who attend publicly managed schools (see OECD [31]).

Figure 2: Gini Index and Stratification



cially the fact that school quality is defined as the mean ability of its students. As in their model, richer households whose children are high skilled pay a tuition so that their offspring can study in a more selective private school. In fact, in an equilibrium with many private schools, there will be stratification by skill and income, with an hierarchy of school qualities, from the lowest level in the public school up to the highest level in the top private school. There are many differences between our model and Epple and Romano's, however. The first is that in their model all children are in school, whereas in ours households can send their children to school or to work, which is more suitable to describe the situation in developing countries. Moreover, Epple and Romano[15] does not explore the consequences of a change in the skill distribution of the student population for the equilibrium number of private schools. Our new theoretical result is that private schools profits depend on the skill heterogeneity among students, so that an increase in the variance of the skill distribution will rise the post-entry profits of the private school. The reason is that students with relatively high skill and income will not stay in the free of charge public school when the student body becomes very heterogeneous and are, therefore, willing to pay for private education. When a private school enters the market, it cream skims the relatively best students and ends up with better peers than the public system.

The model also explores the impact of conditional cash transfers programs on the distribution of students skills in a community. The *Bolsa Familia* Program (BFP hereafter), for example, has been quite successful in reducing poverty and increasing school attendance in Brazil (Glewwe and Kassouf [23], Soares and Sátyro [35]). The BFP brought a new population to the public school system, a large number of new students from very low income classes whose parents had very little formal education. As a consequence, the number of students enrolled in the public school system increased, its average quality fell

and its skill heterogeneity rose, since the new students had very little previous investment in skills from their parents and (in our model) the students' average skill determines school quality.

The model predicts that the rise in the student's skill dispersion brought about by the BFP increases post-entry profits of private schools and therefore the probability of entry. When the private school enters, it attracts the best students away from the public system, leaving it with an even lower quality. Therefore, the impact of the BFP on the total human capital of the economy and its distribution is theoretically uncertain. On the one hand, the beneficiaries of the program increase their level of human capital in comparison with the alternative of not being enrolled in school. The students who migrate to the new private school also improve their human capital, since the private school has a higher quality. On the other hand, the human capital of the students who were previously enrolled in the public school and do not transfer to the private school falls, as the quality of the public schools decreases. Inequality may also increase, since the group who migrates to the private school will become better educated than the group that remained in the public school. Stratification begins at the top.

This paper is at the juncture of several branches of the literature. Despite its importance for a substantial part of the world population, education stratification through the private school system has not received much attention in the economics of education literature so far. Some theoretical models show that private education tends to increase inequality in the steady-state (Glomm and Ravikumar [24] and Zhang, Davies and Zeng [37]). Hoxby [26] uses the religious population in metropolitan areas as instruments for the supply of private schools and finds that private school competition increases test scores and education attainment of the public school students, but has no impact on spending or segregation. Epple, Figlio and Romano [14] finds that the propensity to attend private schools rise with income and ability and that the correlation between income and ability is greater in the public than in the private sector, but increases in the later when public sector expenditure falls. But, no study so far has focused on the drivers of private school entry in the education market.

There is another line of research that examines the relationship between sorting, education and inequality [3] Some papers have focused on the relationship between different education finance systems, stratification and inequality.[4] Fernandez and Rogerson [17], for example, show that if public spending in education is locally financed, in equilibrium families will be stratified by income across communities, with substantial school quality differentiation across them. It has been notoriouly difficult to identifty the models of stratification, however, because of social interactions. In a paper related to ours, Hoxby [25] finds, among other things, that school finance equalization tends to increase enrollments in private schools in penalized districts. We argue that the rapid social changes that have been occurring in Brazil, both in terms of poverty and

[3] See Benabou [6], Durlauf [13] and Fernandez and Rogerson [17], among others.
[4] See Fernandez and Roberson [17] and Fernandez [16] among others.

school attendance, may provide the sort of exogenous variation that is essential to identify the process of sorting and its potential effect on inter-generational mobility and long run inequality.

Our work also relates to the economics of education literature, in line with the papers of Epple and Romano [15] and McLeod and Urquiola [28], among others. In these papers, stratification happens in equilibrium, so that conditional on an income level, the best students attend the private schools. McLeod and Urquiola [28] assume information asymmetry in the post school labor market, so that school reputation works as a signal of the unobserved student quality. Epple and Romano [15] show that with peer effects there will be stratification by skill and income across public and private schools, and also across the private schools. In their model, private schools will offer tuition discounts to the best students, and these discounts may vary with the expenditure in the public system. But, no study so far has examined the impact of changes in inequality in the public school system on private school entry, however.

Entry is the subject of an extensive literature in industrial organization, pioneered by the work of Bresnahan and Reiss [10]. This literature studies the relationship between the determinants of firm's profits and the equilibrium number of firms in a market. Changes in market fundamentals, such as market size and costs affect industry structure. This well established literature, however, does not take into account one specific characteristic of the educational sector: the externality generated by peer students. This sector belongs to a group of markets in which the type of persons who consume the good is important for the utility that other consumers derive from it.[5][6] Their main characteristic is the existence of a positive externality in consumption. In some situations, these markets are characterized by a strong market differentiation, with consumers stratified across qualities in equilibrium. Communities, schools, nightclubs and restaurants are all cases where there exists substantial product differentiation, with groups of consumers associated with certain varieties.

The traditional entry literature does not take into account the full consequences of social interactions, however. Mazzeo [29], for example, estimates an entry model where products are vertically differentiated. The profitability of the firms in these markets crucially depends on the ability to differentiate themselves, with prices being a function of differences in quality. But, in our model quality is no longer a firm's control variable, since it is defined by the consumers choices. This causes a major distinction between entry in a market with vertical differentiation and one with social interactions. In a traditional market, firms enter the market by choosing a position on the quality space. Therefore, changing the distribution of consumers tastes 'far' from where the firm intends to locate will not have any impact on this firm's profitability, since differences in product qualities are fixed. In our model, a school quality is determined by

[5]Other examples are: families prefer to live in communities which share their socioeconomic characteristics; people like restaurants, nightclubs and bars attended by 'cool' people; books and movies become more interesting when more people watch and read them.

[6]Becker [4] calls this process 'social interactions´ and Glaeser and Scheinkman [22] call it 'non-market interaction'.

its students' average skill. Since students choices depends on the equilibrium qualities and prices, changes in market fundamentals that affect this equilibrium will impact firms profits. For this reason, changes in the distribution of consumers 'far' from a firm's location in the product space will affect its profits. The inclusion of low income unskilled students in the students' population induces the entry of a firm that provides a good to the wealthiest share of the students, the private school.

Finally, our paper relates to the growing literature that discusses the relationship between democracy, redistribution and inequality.[7] Some of the papers in this line of research explain why democracy may fail to reduce inequality by exploring the process of capture of the democratic decision making process by a country´s elite (Acemoglu and Robinson [3]). This paper provides an example of how a country´s elite can use private schools to maintain stratification even in the face of a radical change in the political system that leads to an increase in school attendance by the poor. The salient feature of our paper is that this mechanism arises endogenously in equilibrium, without us having to rely on a process of elite capture of the political system.

This paper is organized as follows. The next section setup and analyze a theoretical model of the school market. Section 3 has the empirical analysis. The last section concludes.

2 School Market's Equilibrium

2.1 Overview

Our model consists of households and schools. A household is defined by its income and its child's skill level. There is a continuum of students defined by the joint distribution of income and skill. The household problem is to choose one school among the set of available ones to enroll its child, or send him to work. There is a free of charge public school, which all students can attend, and $N-1$ private schools. We model the private school problem as a two-stage entry game. There are (infinitely) many identical potential entrants in the private school's market. In the first stage, none or some private schools decide to enter the market. In the second stage, the private school sets a price and compete for students with the other schools.

In our model, the average skill of the students enrolled in the school defines its quality, as in Epple and Romano [15], MacLeod and Urquiola [28] among others. The quality of the students defines school quality through peer effects, that is, a student benefits from being among good peers. Behind the peer effect argument is the idea of a production function of education where peer students are inputs in the education's production process. There are several papers that investigate the existence of peer effects in education with mixed results (see Abdulkadiroglu et al [1] for a review). Our empirical results will show that the distribution of students characteristics affects the school's market structure,

[7] See Acemoglu et al [2] for recent survey.

which is an indirect evidence of peer effects on school quality. We also assume the single crossing property on preferences, meaning that high skill students have a higher willingness to pay for education. This assumption could be justified by the findings of Cunha and Heckman [11], which shows that a student's returns to investments in education depends on the previous investments made during her childhood. Therefore, children that received more investments during their earlier childhood will have a higher return to current investment in education, and consequently a higher willingness to pay for better schools.[8]

In an equilibrium with several private schools, stratification by skill and income occurs, as in Epple and Romano [15], but here as a consequence of the single crossing property. The students average skill and income are higher in the best private schools. These averages decrease as we move down the quality ladder. Among all the schools available, the public school has the students with the lowest average skill and income. In other words, the private schools cream skims the best students from the public school. Finally, the poorest households will send their unskilled children to work.

Given that school quality is given by the average skills of its students, it is not profitable for a private school to enter the market when there is no skill dispersion among the students in the public system. Without any skill dispersion, the quality of the private and public schools would be the same and the households would not be willing to pay for private education, as they have the public option for free. Therefore, a higher dispersion of students' skills in the economy increase the potential quality distance between the private and the public schools, implying higher profits for the private schools. The conclusion is that, ceteris paribus, one should expect a greater number of private schools in locations where the students' skill dispersion is wider. This is the intuition behind the main theoretical result in our paper.

The hypothetical situation where all schools (public and privates) enter the market with similar qualities is not a stable equilibrium (see Fernandez [16]). Students will eventually reallocate themselves across the different institutions, converging to the situation where schools are vertically differentiated in quality. As such, the equilibrium price of a given private school is a function of the difference in quality between this school and the public institution. The central difference between our model and traditional models of vertical differentiation - such as Shaked and Sutton [36] and Bresnaham [9] - is that quality is not only a firm's decision variable. Instead, it is also defined by consumers choice.

2.2 The Model

There are M households, each one with income y, with $y \in \left[\underline{y}, \overline{y}\right]$, which is exogenously determined. Each household has a child with skill s, with support $s \in [\underline{s}, \overline{s}]$, where $s = s(y, \varepsilon)$, with $s_1 > 0$ and $s_2 > 0$. Hence, two terms affect the child's skill. The first term states that richer families are capable of investing

[8] Fernandez [16] provides a full discussion of the implications of the single crossing property in education.

more in its child's education and, therefore, the child's skill is an increasing function of the household's income. The second term ε represents factors other than the families income that affect the child's skill such as innate ability, the previous teacher the child had etc. We assume ε is a bounded random variable, with $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$. Given this structure, skill and income are jointly distributed with density $h(y, s)$, which we assume to be continuous.

Each household cares about its consumption and its child's human capital. The family faces a discrete choice problem in which it has to decide whether to send its child to work or to study in a public or in one of the private institutions.[9] The subscript j, with $j = 0, ..., N$, is the number assigned either to the alternative of working ($j = 0$) or to the school attended ($j > 0$). There are N schools in the market, with the public school being represented by $j = 1$. Formally, the household's problem is the following:

$$\max_j \{u(y + w_j - p_j) + \theta_j s : j = 0, ..., N\}.$$

w_j is the child's wage, which is positive if he works ($w_0 > 0$) and zero otherwise ($w_j = 0$ if $j > 0$). p_j is the tuition of school j, where the household does not pay tuition if the child works ($p_0 = 0$) or is enrolled in the public school that is free of charge ($p_1 = 0$). The term $y + w_j - p_j$ is the household's consumption of the numeraire good, with price normalized to 1. Utility function u has the usual properties, and the linear assumption about $\theta_j s$ is for simplicity. The term $\theta_j s$ is the child's human capital. It increases with the child's skill s and the quality of school j, which is measured by the mean skill of its student body θ_j.[10] We assume that $\theta_0 < \underline{s}$, which indicates that the child's human capital is lower if he works, in comparison with the alternatives of attending either a private or a public school. The multiplicative term $(\theta_j s)$ in the household's objective function captures the fact that higher skill students have a higher willingness to pay for education, as their benefits from education are greater, as pointed out it the previous subsection.

Household i is characterized by the pair (s_i, y_i). Given the preferences and the choices the households have, the set of households choosing j, $j \in \{0, 1, ..., N\}$, is:

$$A_j(p., \theta.) = \{s_i, y_i | u(y_i - p_j + w_j) + \theta_j s_i \geq u(y_i - p_k + w_k) + \theta_k s_i, \forall k \neq j\} \tag{1}$$

The share of households choosing option j is given by:

$$S_j = \int\int_{A_j} h(s, y) ds dy \tag{2}$$

[9] In Brazil, as in many countries, according to the law, all children under the age 14 must be enrolled in school. However, the law is not always followed.

[10] As pointed out in the previous subsection, this feature of the model captures the positive externality of studying with good peers.

The mean skill of the students attending school j ($j > 0$) is given by:

$$\theta_j = \int\int_{A_j} sh(s,y)dsdy \tag{3}$$

Note that equations 1 and 3 are simultaneously determined. A_j, the set of households who opt for j, depends on θ_j, which equals the integral over A_j.

There is only one large public school and it offers free admission to all students. The public school problem is not modeled formally. For simplicity, the public school has a passive role in the model in the sense that it does not react to the private schools actions.

We model the private school entry decision as a two stage game. On the first stage the school decides whether to enter or not in the market. On the second stage it sets its price and compete for students with the other schools. In other words, the private schools compete in price. We normalize the private school's opportunity cost to zero, so that its decision rule on the first stage is :

$$\begin{cases} \text{enter} & \text{if} \quad \pi_j \geqslant 0 \\ \text{do not enter} & \text{if} \quad \pi_j < 0 \end{cases}$$

where π_j is its profits, and $j > 1$.

On the second stage, upon entering the market, the school decides its price policy, taking as given the other schools prices, which will depend on its cost structure and demand. It chooses the optimum tuition p_j^* in order to maximize the following profit function:

$$\max_{p_j} p_j S_j M - C(S_j M) - F \tag{4}$$

where F is the sunk entry cost (assumed to be equal to all schools), C is the variable cost (with $C^{'} > 0$ and $C^{''} > 0$), which depends on the number of attending students ($S_j M$).

The Bertrand-Nash equilibrium of the two stage entry game is characterized by $\{p_j, \theta_j\}_{j=0}^{N}$, such that: (i) the households maximize its utility sending its child either to work or to study, taking as given the quality of the schools and the tuitions, (ii) firms maximize profits by choosing whether to enter the market and its tuition upon entering, taking as given other schools tuitions and qualities, with $\pi_j \geqslant 0$ for $j > 2$; (iii) no household wants to change the school of its child; and (iv) the market clears, $\sum_{j=0}^{N} S_j = 1$, that is, the number of children equals the number of students and young workers in the economy.

2.3 Equilibrium with N Schools

We analyze the oligopolistic equilibrium of the school market. The proofs are in appendix A. We start by proving existence.

Proposition 1 *There exists equilibrium with N schools.*

A necessary condition is that the total derivative $\frac{dS_j}{dp_j}$ be negative. Hence, although we expect to see a positive slope is some parts of the demand function, as in Becker [4], equilibrium never happens in these regions. There are multiple equilibria in the school market, but the only stable one is the segregated equilibrium (Benabou [6], Glaeser and Scheinkman [22], Fernandez [16]). The next proposition characterize this equilibrium.

Proposition 2 *In a segregated equilibrium,* $\theta_1 < \theta_2 < ... < \theta_N$ *and* $0 = p_1 < p_2 < ... < p_N$.

Proof. Suppose not. All schools have the same average quality and charge the same prices. This situation is indeed an equilibrium, but it is an unstable one. Let's focus on two schools, k and j, such that, $\theta_k^0 = \theta_j^0$ and $p_k^0 = p_j^0$. Suppose one of them, let's say j receives a positive shock on its quality, such that $\theta_j^1 > \theta_k^0$. Now if both schools charge the same price, all students strictly prefer j. In this situation it is optimal for j to raise its price: $p_j^1 > p_j^0 = p_k^0$. All the students in school j have an utility loss due to the higher price but, by the multiplicativity assumption, the students with higher skill benefit more from the higher average quality. Some of the students with lower ability find it optimal to transfer to the cheaper school k. When this process ceases, the new equilibrium will have $\theta_j > \theta_k$ and $p_j > p_k$. If we apply this argument to all pairs of private schools, we get the complete ordering for the $N-1$ schools. Since the price of the public school is zero, it cannot select the better students by raising the price, as the private school does. So, in equilibrium it can only have the lowest skill students. ■

Recall that a household is characterized by the pair (s, y). We now turn to the proposition that describes how these different types of households are stratified across the alternative private schools, public school and working. Before proceeding, it is useful to define two types of stratification, as in Epple and Romano [15]. We say that stratification by income (SBI) occurs if, for any two households having the same skill, the household that chooses the private school (public school) has a higher income than the one that chooses the public school (to work). Analogously, stratification by skill (SBS) holds if, for any two households having the same income, the household that chooses the private school (public school) has a higher skill than the that chooses the public school (to work). Figure 3 shows how it is an equilibrium in which both types of stratification occur simultaneously. Households with the pair (s, y) in the upper right corner of the figure enroll its children in the private school. Those in the middle send its children to the public school. The ones in the lower left corner opt for the alternative to work.

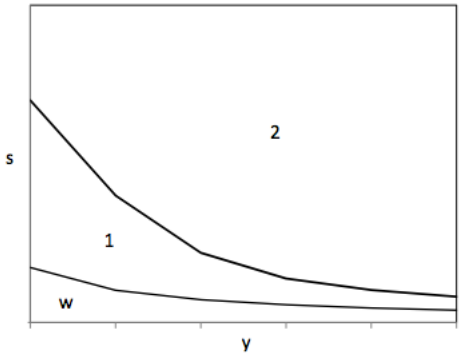
Proposition 3 *In an equilibrium with N schools, SBI and SBS characterizes equilibrium.*

To understand the intuition behind the SBI, suppose two households whose children have the same skill but with different incomes - say y and y', with $y > y'$. Since their children have the same skill, they both benefit equally from

attending school j. However, since utility is a concave function and $y > y'$, it implies that the wealthier (y) household has a smaller decrease in utility when he pays tuition p_j. Therefore, the wealthier household strictly prefers the school j if the less wealthy one is indifferent between schools j and $j-1$.

An analogous argument gives the intuition behind SBS. Suppose two households with the same income but whose children have different skills - say s and s', with $s > s'$. Since they have the same income, their consumption of other goods fall equally when they enroll their children in school j and pay tuition p_j. If the household whose child has skill s' is indifferent between schools $j-1$ and j, the another one strictly prefers school j. The reason is that, as its child have a higher skill, the household benefits more from school j, as its quality is greater than school $j-1$.

Figure 3: School Market Equilibrium



In the next two propositions, we analyze factors that affect private school's profit and market entry.

Proposition 4 *The price charged by any private school is a positive function of the difference in quality between this private school and the public institution. E specifically, for any school j,*

$$p_j = y - u^{-1}\left[u\left(y\right) - \left(\theta_j - \theta_1\right)s\right].$$

The next proposition - our main result - is a comparative statics of the equilibrium we just described. It states that as the skill distribution widens the potential profits of private schools rise.

Proposition 5 *An increase in skill dispersion increases the private schools profits.*

The above proposition states that a rise in skills dispersion implies a higher profitability for the private schools and, as a consequence, entry becomes more

likely. The intuition behind this result is the following. As shown above, the private school tuition increases with the difference in quality between the private and public school. When there is a lot of heterogeneity among the student population, it is easier for the private schools to differentiate themselves from the public school by attracting the students with relatively higher skills. With this differentiation, households whose children have relatively higher skills are willing to pay for a private education and private schools can charge profitable tuition. Ceteris paribus, the greater is the skill dispersion, the higher are private schools profits.

In more homogeneous societies, where students have similar skills, there is little or no room for a private school to enter the market and cream skimming the best students from the public school. In contrast, in more heterogeneous societies private schools are able to attract the best students, have an important differentiation with respect to the public school and charge a high enough tuition, such that entry is profitable.

2.4 The *Bolsa Familia* Program

We now turn to the effects of the BFP on the the school market equilibrium. We note that the results in this section are more general than the specific case of the BFP, but we find it more interesting to carry on the analysis within this well defined context. Before proceeding with the analysis, we briefly describe the program.[11] The BFP is a conditional cash transfer program focused on poor families in Brazil. Families receive a grant from the government and in return have to ensure that their school-aged children are enrolled in a public school and vaccinated. The program aims at reducing poverty in the short-term and increasing human capital among the poor children, through its effects on skill and health. The transfer compensates the households for the opportunity cost of attending a school. If the conditionalities are not met, households may be excluded from the program.

The potential beneficiaries of the program are the households whose children have relatively lower level of skills and are working (the fraction S_0), which are the ones located in the lower left corner in Figure1. The program gives incentives for the households to enroll their children in the public school, so that they do not receive the wage w_0 anymore, receiving the grant from the government as a compensation instead. The effect of the program is to enroll MS_0 students in the public school.

We now use the model results to discuss the effects of this program. The following result has a key role in our empirical analysis, as it states that the *Bolsa Familia* is a source of exogenous variation in our entry model.

Remark 1 *The introduction of the Bolsa Familia program lowers the average quality and increases the skill dispersion in the public school.*

[11] For more information on the program see http://www.mds.gov.br/bolsafamilia.

As the beneficiaries of the program are the households whose children have relatively lower level of skills, when they stop working and enter the public school, the mean skill of the students falls, and the variance increases.

Remark 2 *The profits of the private schools increase when the* Bolsa Familia *program is introduced.*

The children of the households beneficiaries of the program stop working and enter the public school. As they are relatively less skilled, this increases the skill dispersion in the public school. As shown in Proposition 5, the private schools' profits rise with the heterogeneity of student population. The empirical implication is that, ceteris paribus, we are more likely to observe a private school in municipalities where the program penetration is higher.

Remark 3 *The quality of the public school falls even further when a private school enters in the market.*

The above result indicates that one effect related to the presence of a private school in the market is that the quality of the public school falls. The reason is that the equilibrium with a private school is stratified by skill. When the private school enters the market and attracts the relatively best students (with higher skills), it leads unequivocally to a fall in the quality of the public school.

The next remark discuss the effects of the program on the total human capital in the economy and across students.

Remark 4 *The effect of the program on the total human capital in the economy is uncertain.*

The net effect of the introduction of the program on the total human capital in the economy is uncertain, as the program have different effects on different types of students. The human capital of the beneficiaries of the program increases. They stop working and enroll in the public school. By assumption, the student's human capital is lower if he works, in comparison with the alternative of studying. The human capital of those students who were enrolled in the public school before the introduction of the program and stay there after its implementation falls. The reason is that the quality of the public school falls, as discussed above. This decline is aggravated if the private school enters the market. Conditional on enter, those students who leave the public school and move to the private school increase their human capital. In sum, with winners and losers the net impact of the Bolsa Familia on the total human capital in the economy is uncertain and differs across types of students.[12]

[12] In practice, it is possible to say that the human capital of the majority of the students falls. The reason is simple. Empirically, the majority of the students are those who were enrolled in the public school before the introduction of the program and stay there after its implementation. Their human capital fall unequivocally.

3 Empirical Analysis

3.1 Data

The Brazilian education system consists of three main cycles: fundamental (primary and secondary), high school and college. Fundamental education lasts nine years, five years of primary and four of secondary education, while high school and college take three and four years, respectively. All three cycles are offered by private and public schools. Private institutions charge a fee, while the public system is free of charge at all levels. Brazil has 5,563 municipalities spread around the 26 states plus the district capital. Public schools are either managed by the State or by the municipal (local) authority. Public schools offering fundamental education are usually run by the municipal authority, while high schools are generally managed at the State level and colleges/universities by the federal government. There were 13 million students in fundamental education in Brazil in 2011, studying in 146,000 schools, 14% of which were private, caring for 30% of students.

The data we use in this paper come from several sources. The number of private and public schools by municipality come from the School Census, which is carried out annually by the Ministry of Education and has detailed information on every school in the country. In the empirical exercises below we use data on the number of private schools (for profit) offering primary and secondary education by municipality between 2007 and 2011. The information on average household income, education, poverty rate and school-age population for each municipality come from the Demographic Census, carried out decennially by the Brazilian Census Bureau. To obtain yearly values for these variables, we interpolated using the data for 2000 and 2010 by municipality, assuming a constant growth rate over time. Data on test scores, which were used to compute average, standard deviation and the coefficient of variation by municipality, comes from *Prova Brasil*, a standardized national examination carried out every other year since 2005 to assess the quality of public education in Brazil. Once a municipality agrees to participate in the evaluation, all students of urban public schools with more than 20 students take the test. The information on the total number of families receiving the bolsa-familia program in each year comes from Brazilian Social Ministry (MDS). In order to compute the share of the population receiving the bolsa-familia transfer, we multiplied it by four (the average size of a family in Brazil) and divided by the total population of the municipality.[13]

We dropped municipalities with a school-age population (0-15) higher than 15,000, as those cities have dozens of private schools and represent, therefore, more than one education market. Our final dataset has 4,823 municipalities, out of the 5,564 existing in Brazil in 2011, 5,042 of which had a school-age population smaller than 15,000.[14] Table 1 describes the main variables used

[13] We therefore assume that the average size of the families evolved similarly across municipalities between 2000 and 2010).

[14] The main reason for dropping municipalities was the absence of data on test scores, since some municipalities did not agree to take part in Prova Brasil for political reasons

in the analysis, separating municipalities into four different groups: those with private schools throughout the sample period (1,590), those without (2,808), those where there was private school entry (240) and those where there were private schools in 2007 but not in 2011 (185). One can notice that only 33% of municipalities had a private school throughout the sample period. Between 2007 and 2011, entry occurred in 8% of the cities with no private schools in 2007.

One can note from the table that municipalities with private schools were bigger, with a student-age population of 6,321, as compared to 2,304 in the cities without private schools throughout the sample period and around 4,500 in municipalities that changed their private school status between 2007 and 2011. Their population also tends to be richer, with average monthly household per capita income of 439 reais (around 200 US dollars), as compared to 412 in the public school-only cities and 373 in the cities where there was private school entry. Adult education was similar in all types of municipalities, but poverty rates were markedly higher in the cities where there was private school entry, as was the share of the population receiving bolsa familia program. The share of students in school and the various test scores indicators were very similar across the different types of municipalities.

In terms of changes over time, one can note that between 2007 and 2011 the school-age population declined in Brazil, as a result of the demographic transition. It declined less, however, in cities with no private schools and in those where there was private school entry. The first decade of the new century was very good in Brazil in terms of social indicators. Real income rose in all types of cities and so did adult education. The poverty rate declined throughout in the country, by about 5 percentage points, while the coverage of bolsa-familia increased between 6 and 7 percentage points. It is interesting to note that both the decline in poverty and the rise in coverage of bolsa-familia were higher precisely in the municipalities where there was private school entry. Also interesting is the fact that in those cities both the mean and standard deviation of test scores (of public schools students) increased by less, a result that could be the consequence of the new private school attracting the best students of the public system. In the econometric exercises below we will try to uncover the causal relationship between skill dispersion and private school entry.

3.2 Identification Strategy

The theoretical model above predicts that the probability of entry of a private school in a municipality will increase with the size of the school-age population, with income and with the heterogeneity of the students in the public school, proxied empirically by the test scores coefficient of variation (standard deviation divided by the mean). Formally, the main equation to be taken to the data is:

$$Priv_{mt} = \alpha_m + \beta cv_{mt} + \eta lpop_{mt} + \gamma ly_{mt} + \theta X_{mt} + \delta_t + \varepsilon_{mt} \qquad (5)$$

where $Priv_{jt}$ is equal to one if there is at least one private school in the municipality m in time t (and zero otherwise), cv is the coefficient of variation of

the public school students' test scores, *lpop* is the (log of) the total student-age population, ly is the (log of) average household per capita income, X is a vector of other control variables (including the share of adults with at least high school and the poverty rate), δ_t are time dummies to control for common macroeconomic effects, α_m are the municipality fixed effects, to control for unobserved heterogeneity at the city level that is fixed over time and may be correlated with the heterogeneity of student population and with private school entry and ε is a random error.

The main problem with estimating equation 5 using either linear or discrete-choice models is that skill dispersion in the municipalities may be correlated with other unobserved time-varying factors, which would render inconsistent our estimates of β. Therefore, we need an instrumental variable, which should be correlated with the coefficient of variation of test scores but not with the random error. In this paper we argue that the share of the population receiving the bolsa-familia transfers is a valid instrumental variable, since it is correlated with the heterogeneity of the student population, as the program is targeted to the poor families and is conditional on the children of these families going to school. Since school performance is strongly correlated with family background, the children that start attending schools because of the transfers will tend to have a performance in test scores that is lower on average than that of existing students, so that student heterogeneity will rise in the public system. Therefore, the share of families receiving bolsa familia in the municipalities should be positively correlated with the coefficient of variation of test scores, even conditionally on the municipalities fixed effects. This is a testable hypothesis. Moreover, there is no reason why the share of families receiving bolsa-familia should be correlated with other determinants of private school entry, conditionally on the municipalities fixed effects. Our identifying hypothesis, therefore, is that $E[bf'_{mt}\varepsilon_{mt}] = 0$.

The first-stage of our instrumental variable procedure is given by:

$$cv_{mt} = \theta_{mt} + \lambda bf_{mt} + \vartheta lpop_{mt} + \kappa ly_{mt} + \mu X_{mt} + \delta_t + v_{mt} \tag{6}$$

where bf is the share of families receiving the bolsa-familia transfer in municipality m in period t. We implement this estimator by means of the two-stage least squares procedure, which uses only the variation in skill dispersion predicted by the bolsa-famila transfers to identify its effect on private school entry. We implement both stages in a single step and use linear models in all estimation procedures, which are consistent and have the advantage that the estimated coefficients can be directly interpreted as marginal effects.

3.3 Econometric Results

We now present our main econometric results. Table 2 sets out the results of our first-stage regression, using panel data for 2007, 2009 and 2011. In the first column we regress our measure of skill dispersion, the coefficient of variation of public students test scores (8th grade mathematics) in each municipality on the

share of families receiving transfers from the bolsa-familia program, the control variables, time and state dummies, using pooled OLS.[15] We can see that the estimated coefficient on the bolsa-familia share is positive and statistically significant. The size of the coefficient means that a 18% increase in the coverage of bolsa-familia (one standard deviation) increases skill dispersion by about 20% of a standard deviation, a sizable effect. The size of school-age population and adult education are also positively associated with the coefficient of variation, whereas a higher average household income compresses the skill distribution. Column (2) includes the municipality fixed effects to control for unobserved heterogeneity at the municipal level, so that in effect we are now using only the within-municipality variation in the coverage of bolsa-familia over time to identify its effect on skill dispersion. One can see that the bolsa-familia coefficient remains significant. Column (3) repeats the specification of column (2), but uses a long-differences specification, that is, we use the variation of all variables between 2007 and 2011, and the estimated coefficient increases in size. In column (4) we include poverty rate in the econometric model. Since the main aim of the bolsa-familia program is to reduce poverty, we there is a positive correlation between these two variables. Moreover, poorer municipalities have a higher proportion of their kids out of schools and the theoretical model set out in section 2 predicts that skill dispersion in the public school system should be lower in that case. The results show that, indeed, poverty is negatively correlated with skill dispersion and its inclusion decreases the size bolsa-familia coefficient substantially, but it remains statistically significant.

Since the bolsa-familia program is very influential in Brazil and in other developing countries, representing a clear change in priorities that occurred after the return to democracy, it is important to examine its effects more thoroughly. Table 3 does that by reporting estimated coefficients of the bolsa-familia program in long-differences regressions with different dependent variables, controlling also for the change in the school-age population and education of the municipalities.[16] Each row reports the result of a different regression. The second column of the table reports the magnitude of the effect of increasing the coverage of bolsa-familia by one standard deviation (18.7 percentage points) in terms of percentage of standard deviation of the dependent variable. The first row reports the result of using average household income as the dependent variable and shows that increasing bolsa familia coverage by one standard deviation would have a very small impact on average income (only 1% of its standard deviation), despite being statistically significant. In terms of poverty, however, the results of the second row shows that the bolsa-familia program can significantly decrease the poverty rate (by about 10% of a standard deviation). Moreover, since the bolsa-familia is a conditional cash transfer, it should increase school attendance as well. Row (3) shows that it does that in a meaningful way, by

[15] Ferreira and Gignoux [18] discusses the use of the standard deviation of test scores as a measure of education achievement.

[16] We are assuming that the share of bolsa-familia recipients is uncorrelated with other determinants of each dependent variable, which may be a strong assumptions in some of the cases.

raising the share of kids in the 6-17 age group attending school by about 37% of standard deviation. As this kids come from poor families, however, and parental education is so important for school outcomes, bolsa-familia should be also associated with a lower performance in test scores. Row (4) shows that the bolsa-familia coefficient is negative and statistically significant, but the impact is not very high (about 8.5% of a standard deviation). Moreover, it should also increase the standard deviation of test scores, a different measure of skill dispersion from the one we have been using so far. Row (5) shows that this is indeed the case and that the effect is substantial, so that the effects on the coefficient of variation presented in Table 2 do not come solely from its negative impact on the mean scores. Finally, the last row presents the reduced-form effects of bolsa-familia directly on private school entry, without the mediation of skill dispersion and shows that the expansion of the program is a positive predictor of entry.

Table 4 sets out the estimation results of our main equation. The results of column (1), estimated by pooled OLS, with state and time dummies included, show that the skill dispersion, measured by the coefficient of variation of public school students test scores, is positively correlated with the operation of private schools in the municipalities. As predicted by any theoretical entry model, the size of the population and income are also predictors of private school entry. It is interesting to note that the education of the adult population is positively correlated with private entry, as more educated parents should place more value on the quality of education. Poverty rate, on the contrary, is negatively related with entry. All of the variables, therefore, have coefficients estimated with the expected signs. But, skill dispersion in the public system could be correlated with other factors affecting private entry and may be even the result of entry. Therefore, in column (2) we implement our instrumental variable approach, using the share of bolsa-familia recipients in the municipalities as an instrument for the coefficient of variation of test scores. We can see that the coefficient is now about 20 times higher and statistically significant, meaning that higher heterogeneity in the student population increases the likelihood of a private school entering the market.

Column (3) of Table 4 repeats the specification of column (1), but now includes municipality fixed effects. The dispersion coefficient is now negative and statistically significant, possibly reflecting the fact that the entry of a private school may reduce skill dispersion in the public system, by attracting the best students. The results of column (4), which again uses the *bolsa-familia* coverage as instrument for dispersion, show that the estimated coefficient is now positive and similar in size to the specification in column (2), which does not include fixed effects. Columns (5) and (6), repeats the specifications of columns (3) and (4), but now uses a long-differences specification to control for the fixed-effects. The results are very similar, adding credibility to the empirical strategy. The estimated coefficient of column (6) means that increasing the coefficient of variation by one standard deviation (0.027) would increase the probability of entry by 19 percentage points, a sizable impact. In sum, the results show that the variation in the students heterogeneity brought about by changes in the

share of families receiving transfers from the bolsa familia program causes an increase in the probability of private school entry, as predicted by our theoretical model.

Table 5 presents the results of some additional econometric exercises to examine the impact of skill dispersion on private school entry. The first row reproduces the coefficient estimated in column (6) of Table 4 as benchmark. The second row uses reading test scores instead of mathematics as the main explanatory variable. The estimated coefficient is two times higher, but it only marginally statistically significant (p-value equals 0.07). The third row excludes from the sample the 185 municipalities that experienced exits of all existing private schools between 2007 and 2011 and the estimated coefficient of skill dispersion changes very little. Row (4) further excludes from the sample the 1,595 municipalities that had private schools throughout the sample period and one can notice that the size of the estimated coefficient increases, but so does the standard error due to the smaller sample size. In the last row we do as in the evaluation literature and compare the municipalities with a private schools in 2007 with those that did not have one, but were very "similar" to the ones that had. In order to do that we predicted the probability of having a private school in the municipality in 2007 as a function of its population, income, education and poverty and chose from the sample of cities without private schools does that had similar predicted probabilities to the ones that had one (nearest neighbor matching). We ended up with 1,775 observations in the treatment and in the control group and the estimated skill dispersion coefficient, presented in the last row of Table 5, is almost twice the size of the benchmark one.Using this estimate means that increasing the coefficient of variation by one standard deviation would increase the probability of entry by 34 percentage points.

We would also like to know the impact of skill dispersion on the number of existing private schools and not only on whether there exists a private school in the municipality or not. Hence, Table 6 report the results of ordered probit regressions with the same explanatory variables as the previous tables. Column (1) shows that test scores dispersion (coefficient of variation of math scores) in the public system does not have a significant impact on the number of private schools in the city, once we control for other possible determinants, such as the school-age population, average income, education and the poverty rate, all of which have the expected signs. But, we know from out previous results, that we should treat the skill dispersion as endogenous and when we do that, by using the bolsa-familia program as an instrumental variable, we can see that the estimated coefficient is multiplied by a factor of 100 and is now statistically significant. Columns (3) and (4) repeats this exercise using the coefficient of variation in Portuguese test scores instead of in mathematics and the results are pretty similar, with the estimated coefficient on Portuguese being even higher than the mathematics one. Therefore, the results show that skill dispersion not only increases the probability of entry of private schools in the municipality but it also rises the number of private schools in the cities.

4 Conclusions

In 1970 Brazil was under a military dictatorship, there were no direct income transfers to the poor, only 63% of the children aged between 7 and 14 were attending school, the poverty rate was 70% and the Gini coefficient was 0.564. Forty year later, in 2010, Brazil was a stable democracy, a conditional cash transfer program reaching around 44 million poor people was in place, 98% of children were attending school, only 10% of the population were poor and the Gini coefficient was reduced to 0.533. The democratization of the political process in Brazil seems to have been very successful in terms of improving social policies, reducing poverty, increasing school attendance and reducing the country´s historically high levels of income inequality. It seems, therefore, that Brazil's recent history can be considered as a classic example of a change in the political system that, by extending the voting franchise, has led to lower inequality (Meltzer and Richards [30]).

In this paper we show that the rise in the students skill dispersion in Brazilian municipalities, brought about by the expansion of the BFP, led to private school entry. Therefore, the *Bolsa Familia* program, designed to reduce poverty and inequality, had the collateral effect of increasing the entry probability of private schools, and has therefore the potential to generate higher inequality in future generations. Our econometric model used the the participation in the BFP as an instrumental variable for the skill dispersion.

We believe our results could be interpreted more broadly, as an example of how a country´s elite can maintain its relative economic position even in the face of a change in the political system that has led to an increase in school attendance and to the introduction of new social programs. The salient feature of our paper is that this mechanism arises endogenously in equilibrium, without us having to rely on a process of elite capture of the political system.

Future work should examine the impact of private school entry on the test scores of the public schools and on the overall quality and inequality of education in the municipalities. More attention should be paid, moreover, on the impact of the private schools on inequality and intergeneration mobility.

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A Proofs

A.1 Proposition 1

Proof. Consider equation $\theta_j = \iint_{A_j} sh(s,y)dsdy$. In a market with N schools, it defines a system with N equations, one for each school. For a given vector of prices p, we can rewrite it as $\theta_j = H_j(\theta, p)$, where $\theta = (\theta_1, ..., \theta_N)$ and $p = (p_1, ..., p_N)$. Define $H(\theta, p) = (H_1(\theta, p), ..., H_N(\theta, p))$. We can re-write this system as

$$H(\theta, p) = \theta \tag{7}$$

Denote the interval $[\underline{s}, \overline{s}]$ by I. If $s \in I$, by equation (3), $\theta \in I$, a closed and bounded interval. As is usual, we assume that there is a maximum price $\bar{p}$ such that $S(\theta, \bar{p}) = 0$. Denote the price domain $[0, \bar{p}]$ by Π, a closed and bounded interval. $H(.)$ is a continuos function, with $H : I^N \times \Pi^N \to I^N$.

We now turn to the pricing problem. Taking the derivative of equation 4 with respect to p_j we obtain

$$S_j(\theta, p) + p_j \frac{dS_j}{dp_j} = 0.$$

Rearranging terms we have $G_j(\theta, p) = p_j$, where $G_j(\theta, p) = -\frac{S_j(\theta,p)}{dS_j/dp_j}$.

Define $G(\theta, p) = (G_1(\theta, p), ..., G_N(\theta, p))$. We can write the system of N first order conditions as

$$G(\theta, p) = p \tag{8}$$

$G(.)$ is a continuous function, defined as $G : I^N \times \Pi^N \to \Pi^N$.

The system composed by equations (7) and (8) defines the market equilibrium. Define the vectors $\Psi(\theta, p) = (H(\theta, p), S(\theta, p))$ and $\zeta = (\theta, p)$. We can re-write this system as

$$\Psi(\zeta) = \zeta. \tag{9}$$

Since Π and I are closed and bounded intervals, the mapping $\Psi(.)$ is continuous and defined as $\Psi : I^N \times \Pi^N \to I^N \times \Pi^N$, Browers' fixed point theorem applies, so there is a fixed point. ■

A.2 Proposition 3

Proof. In equilibrium, there is a locus of households who are indifferent between being enrolled, say, in school j or school $j+1$. This locus is defined by the following equality:

$$u(y - p_j) + \theta_j s = u(y - p_{j+1}) + \theta_{j+1} s \tag{10}$$

or

$$u(y - p_j) - u(y - p_{j+1}) + (\theta_j - \theta_{j+1}) s = 0.$$

We call this locus $s_j(y)$. Applying the implicit function theorem and rearranging terms, we have

$$\frac{ds_j(y)}{dy} = \frac{\frac{du(y-p_j)}{dy} - \frac{du(y-p_{j+1})}{dy}}{(\theta_{j+1} - \theta_j)} \leq 0 \tag{11}$$

This derivative is the slope of the locus of indiferent students. In order to understand why this is less than zero, first note that in equilibrium with stratification $p_{j+1} > p_j$ and $\theta_{j+1} > \theta_j$. Second, $u' > 0$ and $u'' \leq 0$. It implies that $du(y-p_j)/dy \leq du(y-p_{j+1})/dy$, rendering the numerator negative, and therefore $ds/dy < 0$. The same argument holds for the indifference locus between the public school and the working option. ■

A.3 Proposition 4

Proof. Consider the indifference condition between the public and private school #2:

$$u(y) + \theta_1 s = u(y - p_2) + \theta_2 s \tag{12}$$

Since $u(.)$ is a monotonic increasing function, $u'(.) > 0$, it has an inverse $u^{-1}(.)$, which is also an increasing function. Hence, we can re-write the above equation as

$$p_2 = y - u^{-1}[u(y) - (\theta_2 - \theta_1) s] \tag{13}$$

Therefore, an increase in the term $(\theta_2 - \theta_1)$, increases p_2. Now, consider the indiference condition between schools #2 and #3:

$$u(y - p_3) + \theta_3 s = u(y - p_2) + \theta_2 s \tag{14}$$

Re-writing,

$$p_3 = y - u^{-1}\left[u\left(y - p_2\right) - \left(\theta_3 - \theta_2\right)s\right] \tag{15}$$

Rearranging equation 13, and substituting into equation 15 gives

$$p_3 = y - u^{-1}\left[u\left(y\right) - \left(\theta_3 - \theta_1\right)s\right]. \tag{16}$$

By mathematical induction, we have the following relationship

$$p_j = y - u^{-1}\left[u\left(y\right) - \left(\theta_j - \theta_1\right)s\right]. \tag{17}$$

■

A.4 Proposition 5

Proof. We have shown in Proposition 4 that the price the private school charges is an increasing function of the difference in quality between the public and the private school. It remains to show that (i) an increase in the skill variance increases the difference in quality, and (ii) that profits cannot fall due to smaller market shares in the new equilibrium.

Now we show that an increase in the skill variance increases the differences in quality between the public and private schools. The quality of school j is

$$\theta_j = \iint\limits_{A_j} sh(s,y)dsdy \tag{18}$$

We define the conditional mean skill as

$$\theta_j\left(y\right) = \int_{A_j} sh(s|y)ds, \tag{19}$$

where $h\left(s|y\right)$ is the conditional distribution of skill on income.

Consider now another conditional distribution for skill $h'\left(s'|y\right)$, that is a mean preserving spread of $h\left(s|y\right)$.[17] We can write $s' = s + u$, where u is symetrically distributed as $g\left(u\right)$, with mean zero and positive variance.

The conditional mean skill of the economy under this new distribution is

$$E(s'\left(y\right)) = \int\limits_{\underline{s}}^{\overline{s}} s'h'(s'|y)ds'. \tag{20}$$

Now note that $E(s'\left(y\right)) = E(s\left(y\right))$:

$$E(s'\left(y\right)) = \int\limits_{\underline{s}}^{\overline{s}} s'h'(s'|y)ds' = \int\limits_{\underline{s}}^{\overline{s}} sh(s|y)ds + \int\limits_{-\infty}^{\infty} ug(u)du = E(s\left(y\right)). \tag{21}$$

[17] Such that $h\left(.,.\right)$ second order stochatic dominates $h'\left(.,.\right)$.

Without loss of generality, we consider an equilibrium with three schools: school 1 (public), and schools 2 and 3 (privates), such that $\theta_3 > \theta_2 > \theta_1$. The conditional mean skill of the private school 3 under the new distribution is greater than under the original distribution:

$$\theta_3'(y) = \int_{s_2(y)}^{\overline{s}} s'h'(s'|y)ds' = \int_{s_2(y)}^{\overline{s}} sh(s|y)ds + \int_{s_2(y)}^{\infty} ug(u)du > \theta_3(y). \quad (22)$$

The inequality follows from the fact that the last integral, over the variable u, is not zero anymore, since we are integrating from some positive value to the right (school 3 is the top school). Since it holds for any y, we have that $\theta_3' > \theta_3$. Formally,

$$\theta_3' = \int_{\underline{y}}^{\overline{y}} \theta_3'(y)\, h_y'(y)dy = \int_{\underline{y}}^{\overline{y}} \theta_3(y)\, h_y(y)dy + \int_{\underline{y}}^{\overline{y}} \left(\int_{s_2(y)}^{\infty} ug(u)du \right) h_y(y)dy > \theta_3. \quad (23)$$

Where h_y' is the marginal distribution of y. We can use the same argument to show that the converse holds for the public school:

$$\theta_1' = \int_{\underline{y}}^{\overline{y}} \theta_1'(y)\, h_y'(y)dy = \int_{\underline{y}}^{\overline{y}} \theta_1(y)\, h_y(y)dy + \int_{\underline{y}}^{\overline{y}} \left(\int_{-\infty}^{s_1(y)} ug(u)du \right) h_y(y)dy < \theta_1. \quad (24)$$

Now we need to show what happens to school 2. The sign of the change in θ_2 is undefined, since we do not know the sign of the second integral on the right hand side of the following equation:

$$\theta_2' = \int_{\underline{y}}^{\overline{y}} \theta_2'(y)\, h_y'(y)dy = \int_{\underline{y}}^{\overline{y}} \theta_2(y)\, h_y(y)dy + \int_{\underline{y}}^{\overline{y}} \left(\int_{s_2(y)}^{s_1(y)} ug(u)du \right) h_y(y)dy \quad (25)$$

But what really matters for school 2 profitability is the relative effect of the change in school quality with respect to the public school. We need to show that $\theta_2^{'} - \theta_1^{'} > \theta_2 - \theta_1$. From equations 24 and 25, we have

$$\theta_2^{'} - \theta_1^{'} = \theta_2 - \theta_1 + \left[\int_{\underline{y}}^{\overline{y}} \left(\int_{s_2(y)}^{s_1(y)} ug(u)du \right) h_y(y)dy - \int_{\underline{y}}^{\overline{y}} \left(\int_{-\infty}^{s_1(y)} ug(u)du \right) h_y(y)dy \right] \quad (26)$$

From the definition of the variable u, it follows that the term within brackets is positive.

Last, we need to show that even if the market share of some private schools fall in the new equilibrium with larger skill dispersion, the profit of these schools cannot fall. The equilibrium share of school j is $S_j(\theta_j, \theta_{-j}, p_j, p_{-j})$, where θ_{-j} and p_{-j} are the vectors of other schools qualities and prices. Partial derivatives are: $S_1 > 0$, $S_2 < 0$, $S_3 < 0$ and $S_4 > 0$. Hence, the larger the difference between the private and the public schools' qualities the larger the share of the private school, holding prices fixed.

Therefore, an increase in the difference in qualities allows the private school to do one of three things: (i) increase its price in order to keep its market share constant, (ii) increase its market share by keeping its price constant and (iii) to do a combination of (i) and (ii). All of these options must increase the profit of the private school. ■

B Tables

Table 1: Descriptive Statistics

Variables		No Private Schools	With Private Schools	Private School Exit	Private School Entry
School-Age Population	*Levels*	2,304	6,321	4,350	4,499
		(2,123)	(3,419)	(2,887)	(2,556)
	Changes	-112	-279	-248	-201
		(258)	(402)	(361)	(350)
Average Income	*Levels*	412	439	414	373
		(195)	(210)	(189)	(217)
	Changes	78	67	67	63
		(60)	(40)	(41)	(36)
Share with at least High School Education	*Levels*	0.15	0.19	0.17	0.16
		(0.06)	(0.07)	(0.06)	(0.07)
	Changes	0.050	0.053	0.053	0.053
		(0.19)	(0.19)	(0.024)	(0.21)
Share in Poverty	*Levels*	0.27	0.27	0.28	0.34
		(0.20)	(0.18)	(0.19)	(0.20)
	Changes	-0.054	-0.052	-0.053	-0.058
		(0.024)	(0.026)	(0.028)	(0.027)
Share receiving Bolsa Família Transfers	*Levels*	0.35	0.36	0.36	0.41
		(0.18)	(0.19)	(0.17)	(0.19)
	Changes	0.047	0.046	0.041	0.069
		(0.067)	(0.069)	(0.080)	(0.087)
Share of Students in Schools	*Levels*	0.91	0.91	0.91	0.91
		(0.038)	(0.023)	(0.030)	(0.033)
	Changes	0.026	0.022	0.026	0.024
		(0.025)	(0.018)	(0.021)	(0.020)
Standard Deviation of Math Scores	*Levels*	39.4	40.0	40.1	39.5
		(5.72)	(4.41)	(5.25)	(4.98)
	Changes	2.47	3.06	2.83	2.06
		(6.71)	(4.57)	(5.65)	(5.33)
Mean Math Scores (ln)	*Levels*	5.48	5.46	5.48	5.45
		(0.080)	(0.070)	(0.075)	(0.77)
	Changes	0.022	0.025	0.024	0.020
		(0.057)	(0.043)	(0.051)	(0.050)
Coefficient of Variaton (CV) of Math Test Scores	*Levels*	0.163	0.17	0.169	0.169
		(0.021)	(0.015)	(0.019)	(0.018)
	Changes	0.008	0.009	0.008	0.006
		(0.024)	(0.004)	(0.024)	(0.024)
Number of Municipalities		2,808	1,595	185	240

Table 2: First Stage: Bolsa Familia and Test Score Dispersion

	OLS	Fixed Effects	Long Differences	Long Differences
	(1)	(2)	(3)	(4)
Share with Bolsa Familia	0.022*	0.029*	0.036*	0.024*
	-0,002	-0,004	(0.005)	(0.005)
School-Age Population	0,003*	0,022*	0.021*	0.022*
	-0,001	-0,005	(0.005)	(0.005)
Income	-0.003*	0.017*	0.017*	-0.005
	(0.001)	-0,005	(0.004)	(0.006)
Education	0.030*	0,043*	0.044*	0.057*
	(0.004)	-0,018	(0.018)	(0.018)
Poverty Rate	-	-	-	-0.125*
	-	-	-	(0.018)
N	14,469	14,469	4,823	4,823

Notes: The dependent variable is the coefficient of variation of test scores in the municipality. Robust standard errors are in brackets. Time dummies are included in columns (1) and (2). State dummies are included in column (1). Starred coefficients are significant at the 5% level (*).

Table 3: The Effects of *Bolsa Familia*

	Bolsa-Familia Coefficient	Effect of 1SD increase in Coverage of Bolsa Familia (% SD)
	(1)	(2)
(1) Income	0.037**	1,3
	(0.016)	
(2) Poverty	-0.108**	10
	(0.004)	
(3) Share of Kids in Schools	0.071**	37
	-0,005	
(4) Mean Score – Math	-0.032**	8,5
	(0.011)	
(5) SD Scores – Math	6.79**	23
	(1.25)	
(6) Private School Entry	0.181**	7
	(0.060)	

Notes: Entries in column (1) are the coefficients of the share of the population receiving the bolsa-familia tranfers in of a long-differences regression (2011-2007) that controls for the size of school-age population and education. Robust standard errors in bracket. Each row reports the result of a different regression. Starred coefficients (*) are significant at the 5% level. Entries in column (2) are the simulated effects of an increase in one standard deviation of the bolsa-familia coverage on the variable of interest, measured as a % of standard deviation of the variable.

Table 4: Second Stage: Skill Dispersion and Private School Entry

	POLS	IV	Fixed Effects	Fixed Effects IV	Long Differences	Long Differences IV
	(1)	(2)	(3)	(4)	(5)	(6)
Test Scores Dispersion	0.294*	6.025*	-0.242*	6.328*	-0.241	7.134*
	(0.145)	(1.646)	(0.104)	(2.594)	(0.168)	(3.012)
School-Age Population	0.306*	0,289*	0,116*	-0.052	0.121*	-0.072
	(0.004)	(0,006)	(0,050)	(0.087)	(0.056)	(0.102)
Income	0.243*	0.311*	0.032	0.088	0.041	0.096
	(0.023)	(0.031)	(0.052)	(0.066)	(0.061)	(0.078)
Education	0.622*	0.475*	0.210	-0.162	-0.030	-0.206
	(0.069)	(0.083)	(0.177)	(0.249)	(0.047)	(0.292)
Poverty Rate	-0.346*	-0.280*	-0.327*	0.670*	-0.329	0.800
	(0.061)	(0.067)	(0.172)	(0.437)	(0.194)	(0.508)
N	14,469	14,469	14,469	14,469	4,823	4,832

Notes: The dependent variable is a binary indicator of existence of private schools in the municipality. Test scores dispersion is the coefficient of variation of test scores in the municipality. Robust standard errors are in brackets. Time dummies are included in columns (1) to (4). State dummies included in Column (1) and (2). Starred coefficients are significant at the 5% level (*).

Table 5: Skill Dispersion and Private School Entry - Robustness Tests

(1)	Mathematics	7.13**
		(3.01)
(2)	Reading	14.02*
		(7.82)
(3)	Excluding exits	8.04**
		(2.75)
(4)	No private schools on 2007	10.85*
		(4.47)
(5)	Matched Municipalities	12.56*
		(6.74)

Notes: The dependent variable is a binary indicator of existence of private schools in the municipality. Entries are coefficients of instrumental variable regressions of probability of private school entry on the coefficient of variation of test scores in the municipality. Regressions include controls for school-age population and education. Robust standard errors are in brackets. Starred coefficients are significant at the 5% level(*).

Table 6: Ordered Probit

	Ordered Probit	Ordered Probit - IV	Ordered Probit	Ordered Probit - IV
	(1)	(2)	(3)	(4)
Test Scores Dispersion	0.030	2.505*	0.047	3.269*
	0,032	0.311	(0.029)	0.406
School-age Population	0.070*	0.063*	0.070*	0.062*
	(0.001)	(0.001)	(0.001)	-(0.002)
Income	0.042*	0.069*	0.042*	0.082*
	(0.005)	(0.006)	(0.005)	(0.007)
Education	0.193*	0.133*	0.192*	0.116*
	(0.014)	(0.015)	(0.014)	(0.016)
Poverty Rate	-0.091*	-0.063*	-0.091*	-0.057*
	(0.012)	(0.012)	(0.012)	(0.013)
N	14,469	14,469	14,469	14,469

Notes: Entries are marginal effects of ordered probit regressions. The dependent variable is the number of private schools in the municipality. Test scores dispersion is the coefficient of variation of test scores in the municipality. Robust standard errors are in brackets. Time and state dummies included in all columns. Starred coefficients are significant at the 5% level(*).